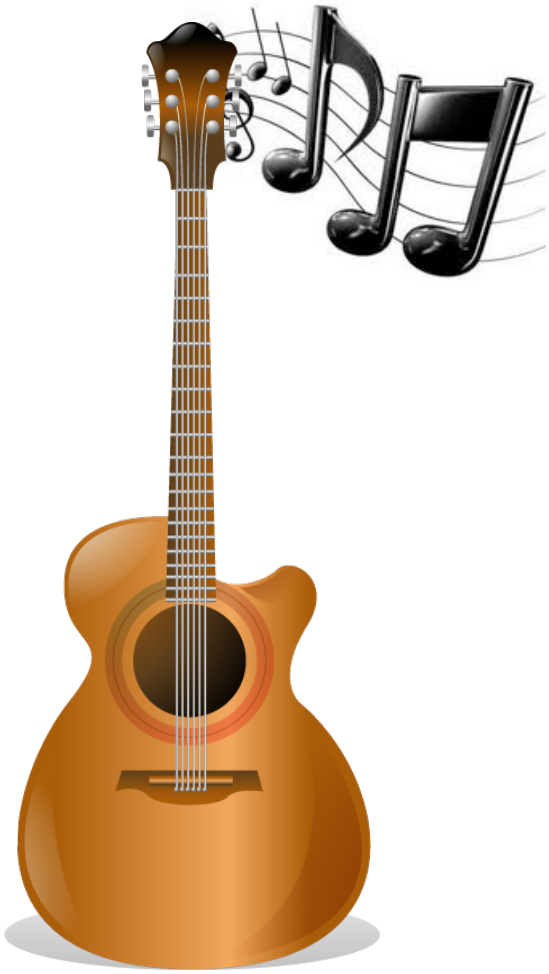


Mathematical Melodies

What Would Life Be Like?

Music and Lyrics by Paul Hanash

What Would Life Be Like?

Sometimes I wonder what life would be like
If I was not a circle, but looked like a kite?
I would have straight lines,
and at least have some sides
But which shape would I be?
I just can't decide, I just can't
Decide!

What would life be like as a straight line
Well I'd go on and on, "Just like time"

What would life be like as a triangle
Well I'd have three sides, "And acute angle"

(Chorus)
But I'm a circle and I'm perfectly smooth
Have no angles like acute or obtuse
I'm unique cause I'm related to pi
And if you want a piece of me you have to go through my radii

What would life be like if I was a square
Well I'd always be right and you'd say, "That's no fair"

What would life be like as a polygon
Well I'd have so many sides you'd have to "Count on and on"

(Chorus)

What Would Life Be Like

Primary/Junior

The Big Ideas

Problem solving is central to learning mathematics. By **teaching through problem solving or teaching about problem solving** students are given numerous opportunities to connect mathematical ideas and to develop conceptual understanding.

Problem solving forms the basis of effective mathematics programs and should be the mainstay of mathematical instruction.

As Greg Tang suggests in his book Mathterpieces: The Art of Problem Solving, "Being good in math requires not only good technical skill but also the ability to apply them in innovative ways."

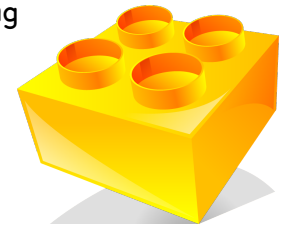
Hats off and pencils on to nourishing creative problem solving in the elementary math class. Math has never been so much fun!

Curriculum Connections

The Mathematical Processes

Problem Solving

Primary (3)



- apply developing problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

Junior (4/5)

- develop, select, and apply problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

Hands On.... Allowing for Choice

By using manipulatives, students have a way to make the abstract real. Teachers have a key role to play in helping students understand the what, where, when and how of math manipulatives.

First, by introducing and demonstrating how to use a variety of math tools in the early grades teachers provide a strong foundation of hands on problem solving strategies. Here, practice makes perfect and once students have had plenty of time with a variety of manipulatives the next step is providing and guiding choice.

By helping students choose the appropriate tool(s) the teacher affords all students the opportunity to choose and test their own strategies, and to construct their own understanding of the problem and its solutions. Reminding students of some of the manipulatives that may be useful (interlocking cubes, pattern blocks, or counters) but not suggesting that they are the only manipulatives available encourages students to choose and experiment with strategies that they see fit.

Inaugural Voyage

Teaching Through Problem Solving – How to Plan?

A 4 Step Process to help teachers design good open ended questions for student problem solving

Step 1: Identify the concept (rather than a skill or a procedure) that you want students to develop.

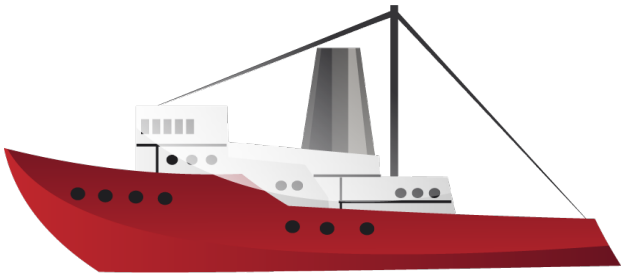
Step 2: Locate the lesson or task in textbooks or professional resources that address the concept.

Step 3: Ask the following questions about the lessons or task:

- Does the problem focus on conceptual understanding rather than just on the procedures?
- Is the mathematics the central focus of the lesson or task?
- Does the lesson or task require justifications and explanations for answers or methods?
- Is there more than one method for arriving at a solution?

Step 4: If the problem does not meet the criteria in step 3, revise the problem.

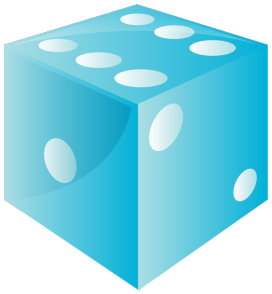
This process comes from the Ontario Ministry of Education' publication entitled A guide to effective instruction in mathematics kindergarten to grade 6: Volume 2 Problem Solving and Communication.



Let's Play a Game...

Mathematical games develop mathematical communication as students explain and justify their moves to one another. In addition, games can motivate students and engage them in thinking about and applying, concepts and skills. Games give pupils an opportunity to communicate their ideas and justify their thinking.

In using games, the teacher plays an important role in encouraging pupils to explain their thinking in keeping them focused on mathematical ideas. Asking them to explain and justify their moves during a trial round of the game played as a whole class demonstrates the type of thinking and communicating that is important for students to use later when they play the game in pairs.



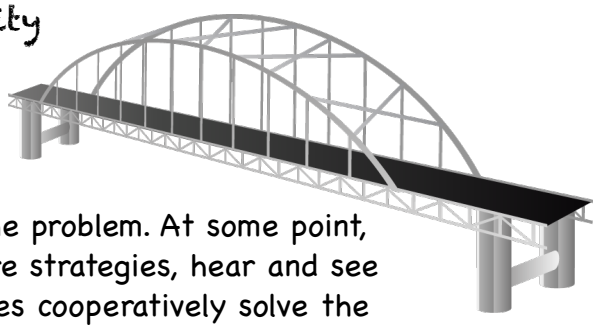
When using games as a learning tool teachers play a pivotal role in asking probing, open questions and creating a classroom environment that encourages experimentation. The focus must be on cognitive processes rather than on the correctness of final outcomes.

Which game?

When considering what games to use it is vital that the context in which they are to be used is considered. The thinking behind each game should be analyzed and matched to the learning objective that are to be met.

Looking at some of the questions which pupils should ask themselves when starting to play a game, and putting them under a mathematical heading gives a good idea to the higher order skills involved.

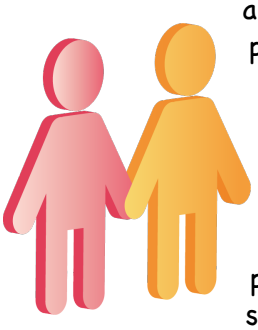
Cross Curricular – The Corner of Curiosity



Creating a place in the classroom where interesting problems can be posted is a way to provide further opportunities for students to practice their problem solving skills and connect math with other subject areas. Here students are given time throughout the day or the week to visit the corner and solve the problem. At some point, the whole class is brought together to discuss the problem, share strategies, hear and see strategies used by other students, evaluate solutions, and at times cooperatively solve the problem.

Character Education

Persistence is an important quality in life and an important quality in math problem solving.



Persistence is the ability to stay with a task for a reasonably long period of time. Children learn persistence when they are successful at a challenging task. The art in building persistence is in offering a task that is just challenging enough, but not overwhelming.

Teachers can try the following strategies to promote persistence:

- Develop a vocabulary for persistence. Terms like work hard, hang in there need to part of the everyday vocabulary.
- Point out to children when they stick at a task.
- Highlight times when your students and you have experienced success by hanging in.
- Talk about hard work. They need to understand that to be successful they need to do things that are NOT fun or easy.
- Praise effort and stick-to-it-ness daily!

Book Connection:

Marvelous Mattie: How Margaret E. Knight Became and Inventor by Emily McCully

The author emphasizes Knight's remarkable accomplishments and persistence during an era in which many believed "that women's brains were inadequate for inventing."

Multi-Media

Websites:

Illuminations is a fantastic website hosted by the NCTM which provides resources, activities and lessons on all strands in the Ontario Curriculum. For creative problem solving check out:

- K-2 Get the Turtle to the Pond http://illuminations.nctm.org/LessonDetail.aspx?ID=L396
- 3-5 Every breath you http://illuminations.nctm.org/LessonDetail.aspx?ID=L243

Learn 360

Has over 25 clips related to mathematical problem solving. For example:

- Math problem solving strategies using manipulatives: Using a model/estimating and checking
- Math problem solving strategies using manipulatives: Patterning and Classifying

SmartBoard: Problem Solving Steps

http://exchange.smarttech.com/search.html?q=math+problem+solving

If I Built a Car!

Using Chris Van Dusen's book to teach Polya's 4 step problem solving model.

Supplies: If I Built a Car by Chris Van Dusen

Why a problem solving model?

Helping students develop a mental model for problem solving is crucial. Polya's four-step model provides a framework for helping students to think about the question posed before, during, and after the problem-solving experience. Typically Polya's four-step model is recommended for grade 3 and above because young students become too focused on the model and less concerned or connected with the mathematical concepts and sense making of the problem. Teachers in the early primary grades who are aware of the model can use it to guide his or her questioning and prompting during the problem-solving process. This guidance will provide students with a valuable skill that is generalizable to other problem-solving situations, not only in mathematics but in other subjects as well.

The Lesson:

"Have you ever dreamed of building your own car? I know I have. Today we are going to read If I Built a Car by Chris Van Dusen. When you listen to this book I want you to play close attention to what steps Jack, the main character in this book, takes to build his car." Read the book out loud with expression.

After reading the book have a short discussion focussing on two things: Jack's ideas and the method he used to plan. Hone in the discussion by showing the first page in the story where Jack is discussing his car idea with his father. "The book starts by showing Jack understanding the problem. What is the problem?" (He wants to build a car that fits his needs). So the first step that Jack takes is Understand the Problem. I am going to keep track of these steps on the board. Write **Step 1: Understand the Problem** on the board.

"What does Jack do next?" Discuss, then go to the next page which shows Jack on the floor planning. What is Jack doing on the floor? "So in the second stage of solving his problem he makes a plan." Write **Step 2: Make a plan.** Discuss strategies that Jack used to make his plan (drawing, making a diagram, using models, researching other cars).

"Then, Jack carries out his plan. Go through some of the pages in the book highlighting Jack's unique ideas for his car (swimming pool, robot, it can submerge, etc). Write **Step 3: Carrying out the plan** on the board.

"Finally, let's look at one of the very last pages where Jack is in the car and talking with his father about his idea. He seems to be reflecting on his ideas." Write **Step 4: Looking Back and Reflecting on the solution** on the board.

While standing next to your list of steps, state, "This 4 step plan is an excellent model for problem solving. It can be used in art, writing, science and **even math**. Let's read through all 4 steps together." The class reads the list together. "These steps will be a helpful guide to you when you solve a problem."

"Please note that problem solvers do not always follow the stages of the model in a lockstep fashion; they often need to go back and forth between stages in order to understand the problem, try out strategies, and find appropriate solutions. We see this in the picture book we just read. I want to highlight two last things from the book. There is one sentence that is very important and is found at the beginning (sorry this book has no page numbers). Highlight the sentence "Constantly analyze, tweak and refine" in the book using sticky notes. Underline each word and explore them with your class. "What do these words mean? Discuss briefly. "Sometimes in life and in math when we are problem solving we will need to go back and forth between these stages, testing and trying our ideas out until the solution fits. Jack, knows this too because what do we see on the last page of the book?" (he is back at the planning stage). He is refining his ideas!

Extensions:

- Have students answer the question, " If I built a Car.' Using the problem solving model have your students understand, plan, and build their own model of something. While testing their model may be hard to do, it does provide a chance to highlight the fact that it is not always necessary to take each step in the 4 step model.
- Take time to go over each step with further points and examples. The chart on the right is sure to help!
- Focus frequently on Step 2 by introducing and practicing a variety of problem solving strategies. These strategies are best explored by students incidentally, within the context of solving daily problems, rather than through direct instruction about the strategies themselves. The eworkshop website created by the Ontario Ministry of education provides some great posters of a variety of problem solving strategies. Here is the link - http://www.eworkshop.on.ca/edu/pdf/Mod18_sample_posters.pdf. They also provide a useful lesson planning template for math problem solving - check out - http://www.eworkshop.on.ca/edu/pdf/Mod18_lesson_template.pdf.

Four-Step Problem-Solving Model

Phases of the Four-Step Model	Implications for Teaching
• Understanding the problem	Students should be encouraged to think and talk about the problem and to restate it in their own words before they go to manipulatives or to paper and pencil.
• Making a plan	Students should be guided to develop a plan. They should realize that all plans are tentative and may be changed throughout the process. They can consider strategies they might use. Suggestions such as looking at the classroom strategy wall might be helpful. It is not necessary for students to record the plan in writing.
• Carrying out the plan	At this phase, students are carrying out their plan and using strategies such as drawing a picture or working with manipulatives. Teacher prompting at this time should focus on questions that elicit greater understanding but should avoid inadvertently solving the problem for the student. Perseverance at this stage should be encouraged. Suggestions to help the student become unstuck can be provided – for example: "Ask Natalie for an idea." "Refer to the strategy wall for another approach." "Can you think of a problem that is similar to this?"
• Looking back and reflecting on the solution	During this "getting back together" phase, it is crucial that students share their ideas in the large group. As a result of the sharing, they can begin to discern that a variety of strategies can be used. They also begin to evaluate critically which strategy works best for them (e.g., is most efficient, is easiest to understand). Teachers should encourage students to discuss what they have learned through the problem-solving experience and to pose new problems that are related to the one just solved.

Picture This!

More Literature Links for creative problem solving involving ....geometric shapes and wild imaginations!

Fiction:

The Village of Round and Square Houses by Ann Grifalconi (2008)
Tos is an actual village, on the side of an inactive volcano in Central Africa. It is the village in the book's title, where women dwell in round thatch-roofed huts and men live in square ones. This book tells the story of how the houses were so designated, in the aftermath of the volcano releasing its full fury on the village.

It looked Like Spilt Milk by Charles Shaw(1993).
The white shape silhouetted against a blue background changes on every page. Is it a rabbit, a bird, or just spilt milk? Children are kept guessing until the surprise ending and will be encouraged to improvise similar games of their own.

Whose Shadow is This?:A Look at Animal Shapes - Round, Long and Pointy by Claire Berge (2006)
Examines a variety of different animal shadows including shadows produced from a hummingbird, a gibbon, a reindeer, a camel, a porcupine, a giraffe, and an ant.

The Straight Line of Wonder by Mem Fox (1997)
Despite the admonitions of his friends, a straight line enjoys expressing himself by twirling in whirls, pointing his joints, and creeping in heaps.

Circle Dogs by Kevin Henkes (1998)
Circle dogs live in a square house with a square yard and spend a busy day eating circle snacks, digging circle holes, and sleeping.

If I Built a Car by Chris Van Dusen (2005)
Young Jack is giving an eye-opening tour of the car he'd like to build. There's a snack bar, a pool, and even a robot named Robert to act as chauffeur. With Jack's soaring imagination in the driver's seat, we're deep-sea diving one minute and flying high above traffic the next in this whimsical, tantalizing take on the car of the future.

Mathterpieces:The Art of Problem Solving by Greg Tang (1997)
In this book Greg Tang underscores the importance of four basic rules in problem-solving. Keeping an open mind, looking for unusual number combinations, using multiple skills (like subtracting to add) and looking for patterns. In Mathterpieces, Tang continues to challenge kids with his innovative approach to math, and uses art history to expand his vision for creative problem-solving.

The Greedy Triangle by Marilyn Burns (1995)
Dissatisfied with its shape, a triangle keeps asking the local shapeshifter to add more lines and angles until it doesn't know which side is up.

All content for Picture This was provided by Novelist (http://www.ebscohost.com/novelist/).

References

Ontario Ministry of Education. A guide to effective instruction in mathematics kindergarten to grade 6: Volume 2 Problem Solving and Communication. Retrieved August 12, 2011 from http://www.eworkshop.on.ca/edu/resources/guides/Guide_Math_K_6_Volume_2.pdf

Ontario Ministry of Education. (2005).The Ontario curriculum grades 1-8: Mathematics. Retrieved September 8, 2010 from http://www.edu.gov.on.ca/ eng/curriculum/elementary/math18curr.pdf

Reys, R., Lindquist, M., Lambdin, D., Smith, N., & Colgan, L.(2010). Helping Children Learn Mathematics. Toronto: John Wiley & Sons Canada, Ltd.

Small, M. (2009). Good questions: Great ways to differentiate mathematics instruction. (2009). New York: Teacher's College Press.

Van de Walle, J.A., Folk, S., Karp, K., & Bay-Williams, J. (2011). Elementary and middle school mathematics: Teaching developmentally. Toronto: Pearson.