## Probability

## Activity 4 - The Handshake Problem

Imagine that you are having a birthday party. You invite 10 of your friends: Amy, Bobby, Cindy, David, Ellen, Frederick, Gertrude, Harry, Jerry and Kate to attend. Because you and your friends are all very nice and friendly people, you naturally want to shake hands with each other. If everyone at your party, including you, shakes hands with everybody else once, how many handshakes will there be in total? If two people shake hands with one another, we will have one handshake. In order to solve this problem, you need to figure out how many possible combinations of two people shaking hands can there be at your party. This is hard to do right away for all 11 people, so start with smaller numbers and then gradually figure the numbers out for 11 people.

To make writing notation simpler, let:

$$
\begin{gathered}
\mathbf{A}=\text { Amy } \quad \mathbf{B}=\text { Bobby } \quad \mathbf{C}=\text { Cindy } \quad \mathbf{D}=\text { David } \quad \mathbf{E}=\text { Ellen } \quad \mathbf{F}=\text { Frederick } \\
\mathbf{G}=\text { Gertrude } \quad \mathbf{H}=\text { Harry } \quad \mathbf{I}=\text { You } \quad \mathbf{J}=\text { Jerry } \quad \mathbf{K}=\text { Kate }
\end{gathered}
$$

Let's try to organize the number of handshakes in the table below:

| People <br> Shaking <br> Hands | Number <br> of People | Combinations of People Shaking Hands | Number of <br> Handshakes |
| :--- | :---: | :--- | :---: |
| $A$ | 1 | 0 (you need at least two people for a handshake | 0 |
| $A, B$ | 2 | $A B$ | 1 |
| $A, B, C$ | 3 | $A B, A C, B C$ | 3 |
| $A, B, C, D$ | 4 | $A B, A C, A D, B C, B D, C D$ | 6 |
| $A, B, C, D, E$ | 5 | $A B, A C, A D, A D, A E, B C, B D, B E, C D, C E, D E$ | 10 |
| $A, B, C, D, E, F$ | 6 |  |  |
| $A B C D E F G$ | 7 |  |  |

## Can you believe THIS is math?

## Probability

## Activity 4-The Handshake Problem - continued

Another way to help understand and visualize this problem is by drawing circles and lines inside them to show the number of handshakes:

If $A$ and $B e$ are shaking hands, then we draw two points on the circle and joun them with a line. This line tells us that the number of handshakes in this situation is equal to one


Similarly, if A. B and C are shaking hands, the diagram is: number of lines = number of handshakes
= three


For $A . B, C$, and $D$ the diagram is:
Number of lines $=$ number of handshakes $=$ six


The diagrams continue in a similar way, increasing the number of dots and lines to count the handshakes.

## Activity Questions:

- Complete the table on the previous page for six and seven people.
- Make two more diagrams (like the ones shown) for five people and six people shaking hands
- Do you see a pattern in the number of handshakes (That is numbers in the 4th column in the table)?
- Figure out the number of handshakes for eight, nine, 10 and 11 people according to your pattern.


## Can you believe THIS is math?

